Goal-oriented adaptivity in complementary geometric formulations for electrostatics

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An innovative approach has been recently proposed for the voltage breakdown prediction in high voltage systems, insulated by large vacuum gaps. This approach is based on complementary geometric formulations for electrostatics coupled to the analytical solution of the equations of motion for charged particles. In this paper a goal-oriented local mesh refinement technique is introduced, which allows to increase the rate of convergence of the solution, enabling an effective voltage breakdown prediction also in large scale systems with complex geometries.

***Index Terms*—electrostatics, complementary formulations, charged particles, mesh refinement**

I. Introduction

AN INNOVATIVE technique has been recently proposed by the authors [1], [2], [3] for the voltage breakdown prediction in high-vacuum devices used extensively in many research and industrial areas. The proposed approach has been implemented in the numerical code CAFE [2], that uses two complementary geometric formulations for electrostatics and a suitable post processing tool for the tracing of charged particle trajectories, as required by the probabilistic model adopted.

In this paper, we propose a *goal-oriented* automatic mesh adaptivity cycle [4] which allows to improve the rate of convergence of the solution and enables an effective voltage breakdown prediction also in large-scale systems (tens of millions of unknowns) with complex geometries.

II. Voltage breakdown prediction model

The code CAFE [2] is used to solve electrostatic problems in 2D and 3D domains with complementary formulations. Here, we refer to the version based on the electric scalar potential ($\mathbf{V}$) in combination with the mixed-hybrid formulation ($\mathbf{H}$) presented in [5]. The results from the complementary formulations are used as a robust error estimator for adaptively refine the mesh and to have a reliable control on the accuracy of the solution [6].

The voltage breakdown prediction model is based on a statistical approach which requires the knowledge of the electric field in the whole domain to calculate the trajectory of each charged particle. By adopting first order elements, the electric field E inside each cell is uniform and the charged particle is subjected to a uniformly accelerated motion, thus allowing the calculation of the trajectory in closed form as described in [2].

A. Goal-oriented automatic mesh adaptivity

The numerical error of the finite element approximation is estimated in terms of a quantity of interest rather than the classical electrostatic energy as in classical complementarity [7]. The quantity of interest is in our application the electric field computed in the start and end point of each particle trajectory (one per element of the surface mesh used to cover electrode 1), given that the discharge probability function depends on the values calculated for each trajectory as

$$W = \Delta V \, E(P_1) \, E(Q_2)^{2/3} > W_S \tag{1}$$

where $E(P_1)$ denotes the norm of the electric field at the starting point P_1 (responsible for clump charging at electrode 1), $E(P_2)$ denotes the norm of the electric field at the destination point P_2 (responsible for the vapour bubble ionization at electrode 2), ΔV is the voltage between the two electrodes and W_S is a threshold value.

Following the same philosophy behind the constitutive error [6], the proposed error indicator is built from the mismatch in the quantity of interest produced for a given mesh by the two complementary formulations. These formulations, in fact, give the same results when the mesh grain is pushed toward zero, thus refining the "guilty" elements (i.e. the elements with the biggest mismatch in the quantity of interest) cannot fail to improve the accuracy of the simulation [7].

III. Numerical results

Two benchmark electrostatic problems are firstly considered for assessing the robustness and reliability of the proposed particle tracing algorithms.

The first benchmark consists in the calculation of the trajectory of a charged particle in the 2D axysimmetric configuration shown in Fig. 1-left. In this planar central force field, the total energy W (sum of kinetic and electric potential energy) and the angular momentum L are conserved along the charged particle trajectory. Fig. 2 shows the (normalized) values of W and L along the trajectory calculated with the two methods. Tab. I summarizes the results in terms of global errors (max and rms).

In the second benchmark, two concentric spherical shells electrodes are considered. The trajectory of a charged particle in this configuration has an analytic solution (an ellipsis, or part of it) and allows a point-wise comparison of the trajectory. The problem is solved in CAFE with a three-dimensional

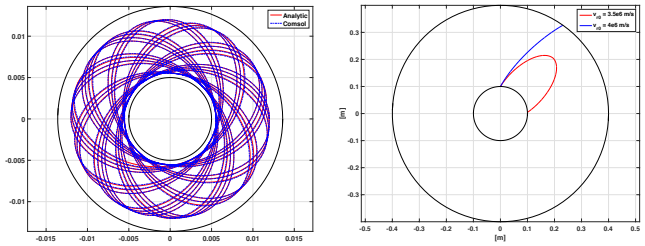

Fig. 1. Left: cylindrical system (r_1 = 1cm, r_2 = 2.7cm, ΔV=1kV). Trajectory calculated with analytic method (CAFE, red) and numerical integration scheme with tolerance ϵ=10^{-6} (Comsol, blue). Right: spherical system (r_1 = 10cm, r_2 = 40cm, ΔV=100kV). Trajectories calculated with CAFE: starting point at r_1 with tangential velocity $v_t = 2.210^6 m/s$ and two alternative values of the radial component: $3.5\,10^6 m/s$ (red) and $4\,10^6 m/s$ (blue).

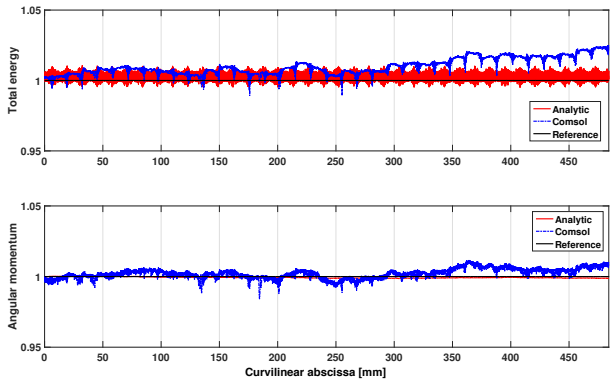

Fig. 2. Total energy W and angular momentum L, as a function of the curvilinear abscissa along the trajectory, calculated with analytical approach (red) and numerical integration scheme (blue) implemented in Comsol (ϵ=10^{-6}).

model (the numerical domain is covered by $\approx 10^6$ tetrahedra) without imposing any symmetry condition. In Fig. 1-right two representative trajectories are shown: in both cases, the maximum discrepancy between the trajectory computed in CAFE and the reference one is of the order of $1mm$.

Then, the proposed automatic mesh refinement procedure is applied to the geometry shown in Fig. 3-left which represents a schematic view of a typical Vacuum Circuit Breaker (VCB). A three-dimensional solution is obtained in CAFE (Fig. 3-right) and validated against the 2D axysimmetric solution obtained with the electric vector formulation ($\mathbf{P}$) introduced in [8].

We remark that in the scalar potential formulation the electric field is discontinuous between elements sharing a face even if they have the same material parameter. It is therefore impossible to construct a streamline crossing these two elements with a classical method based on particle tracking [9]. That is why the discharge probability produced by the scalar potential formulation should be always considered less accurate than the one produced by the mixed-hybrid formulation, which also motivates the interest in the latter formulation.

TABLE I
ERRORS ON NORMALISED ENERGY AND ANGULAR MOMENTUM VALUES

	W - error		L - error	
	rms	max	rms	max
CAFE algorithm (analytic)	0.00379	0.01082	0.00191	0.00352
Comsol ($\epsilon = 10^{-4}$, default)	0.07969	0.22930	0.07002	0.18316
Comsol ($\epsilon = 10^{-5}$)	0.02340	0.10716	0.02404	0.09237
Comsol ($\epsilon = 10^{-6}$)	0.01099	0.02516	0.00382	0.01587

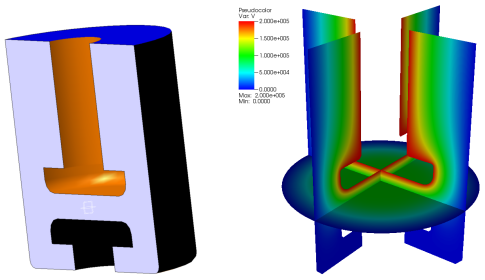

Fig. 3. Left: schematic view of a typical VCB. Right: Cut view of the 3D electric potential distribution obtained in CAFE (electrode gap g=10mm, applied voltage ΔV=200kV).

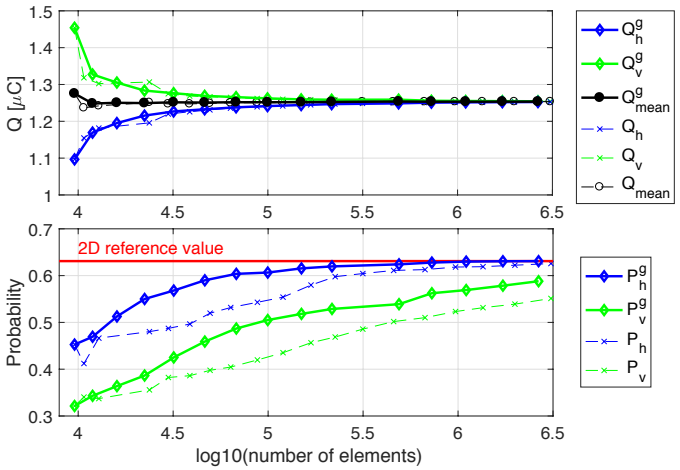

Fig. 4. Rate of convergence of the electric charge deposited on the positive electrode (top) and voltage breakdown probability (bottom). The results of both formulations ($\mathbf{V}$, $\mathbf{H}$) calculated by the new mesh adaptivity scheme (solid) are compared to those calculated by the previous one (dashed).

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